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a)

$V = \pi r^2 h$

as $V = 10$

hence $10 = \pi r^2 h$

$h = \dfrac{10}{\pi r^2}$

$\therefore$ as $A = 2\pi r^2 + 2\pi r h$

sub in h

$A = 2\pi r^2 + 2\pi r\left(\dfrac{10}{\pi r^2}\right)$

$= 2\pi r^2 + \dfrac{20}{r}$

ii) $A = 2\pi r^2 + \dfrac{20}{r}$

$\dfrac{dA}{dr} = 4\pi r - \dfrac{20}{r^2}$

$\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{40}{r^3}$

$\therefore$ A has a min. value as $\dfrac{d^2A}{dr^2}$ is > 0 for all values r as the equation is all positive and r must be positive as a neg. lengths is meaningless.

$\therefore$ min. value occurs when $\dfrac{dA}{dr} = 0$

$\therefore \quad 4\pi r - \dfrac{20}{r^2} = 0$

$4\pi r^3 - 20 = 0$

$4\pi r^3 = 20$

$r^3 = \frac{5}{\pi}$

$r = \sqrt[3]{\frac{5}{\pi}}$

$= 1.17m$ (d.p.)

b) i) $LHS = \sec^2 x + \sec x \tan x$

$= \cancel{\frac{1}{\cos^2 x} + \sec x}$

$= \frac{1}{\cos^2 x} + \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$= \frac{1+\sin x}{\cos^2 x}$

$= RHS$

ii) as $\sec^2 x + \sec x \tan x = \frac{1+\sin x}{\cos x}$

$LHS = \frac{1+\sin x}{\cos^2 x}$

$= \frac{1+\sin x}{1-\sin^2 x}$

$= \frac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(1-\sin x)}$

$= \frac{1}{1-\sin x}$

iii) $= \int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sec^2 x + \sec x \tan x \; dx$

$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{4}}$

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$= \left(\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\right) - \left(\tan(0)\ \sec(0)\right)$

$= 1 + \frac{1}{\sqrt{2}} - 0.1$

$= 1 + \frac{1}{\sqrt{2}}$

$= \frac{2+\sqrt{2}}{2}$ units

~~d)~~

c) $A_1 + A_2 = A_3$

$= 2 \text{ units}^2$

$\therefore A_3 = \int_a^1 \frac{1}{x} + \int_1^b \frac{1}{x} \quad dx$

$\therefore A_1 = \int_a^1 \frac{1}{x} \, dx$

$1 = [\ln x]_a^1$

$1 = \ln 1 - \ln a$

$1 = -\ln a$

$-a = +e$

$a = -e$

$\therefore A_2 = \int_1^b \frac{1}{x} \, dx$

$1 = [\ln x]_1^b$

$1 = \ln b - 0$

$e = b$