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3. a)

$L = \pi r^2 h = 10$

$h = \frac{10}{\pi r^2}$

$A = 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi r\left(\frac{10}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{20}{r}$

ii) $A' = 0$

$= 4\pi r^{1} - 20r^{-2} = 0$

$4\pi r - \frac{20}{r^2} = 0$

$2\pi r = \frac{20}{r^2}$

$r^3 = \frac{20}{2\pi}$

$r^3 = \frac{10}{\pi}$

$r = \sqrt[3]{\frac{10}{\pi}}$

test $A'' = 4\pi + 40r^{-3}$

$A''\left(\sqrt[3]{\frac{10}{\pi}}\right) = 4\pi + \frac{40}{\left(\frac{10}{\pi}\right)}$

$= +ve$

$\therefore$ min at

$r = \sqrt[3]{\frac{10}{\pi}}$

b, i)

$\sec^2 x + \sec x \tan x = RHS$

$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$= \frac{1 + \sin x}{\cos^2 x} = RHS$

ii)

$\frac{1 + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x}$

$\frac{1 + \sin x}{(1 + \sin x)(1 - \sin x)}$

$= \frac{1}{1 - \sin x} = LHS$

iii) $\int_0^{\pi/4} \sec^2 x + \sec x \tan x \, dx$

$= \left[\tan x + \sec x\right]_0^{\pi/4}$

$= \left[\tan \pi/4 + \sec \pi/4\right)$

$-\left(\tan 0 + \sec 0\right)$

$= \left(1 + 1/\sqrt{2}\right) - \left(0 + 1/1\right)$

$= 1/\sqrt{2}$

(c)

$$\int_1^a \frac{1}{x}\,dx + \int_b^1 \frac{1}{x}\,dx$$

$$= \left[\ln x\right]_1^a + \left[\ln x\right]_b^1$$

$$= \ln a - \ln 1 = \ln 1 - \ln b$$

$\ln a = 1$ $\quad$ $\ln b = -1$

$a = e^1$ $\quad$ $b = e^{-1}$

Additional writing space on back page.