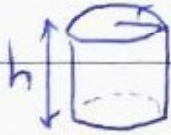


Start here for

Question Number:

5

a) i)



$$V = 10 \text{ m}^3$$

$$V = \pi r^2 \times h \quad \pi r^2 \times h = 10$$

$$A = SA$$

$$h = 10 - \pi r^2$$

$$A = 2\pi r^2 + 2\pi r h \quad \text{show } A = 2\pi r^2 + \frac{20}{r}$$

$$A = 2\pi r^2 + 2\pi r(10 - \pi r^2)$$

$$= 2\pi r^2 + 20\pi r - 2\pi^2 r^3$$

$$= 2\pi r^2 + 20\pi r$$

$$A = 2\pi r^2 + \frac{20}{r}$$

$$\text{ii) } 2\pi r^2 + \frac{20}{r} \quad \frac{d}{dr} \left(2\pi r^2 + \frac{20}{r} \right) \quad u = 2\pi r^2 \quad v = \frac{20}{r} \quad \frac{d}{dr} \left(\frac{u}{v} \right) = \frac{u'v - uv'}{v^2}$$

$$\frac{dA}{dr} = 4\pi r + \frac{-20}{r^2} \quad u' = 4\pi r \quad v' = -\frac{20}{r^2} \quad \frac{d}{dr} \left(\frac{u}{v} \right) = \frac{u'v - uv'}{v^2} = \frac{4\pi r \cdot \frac{20}{r} - 2\pi r^2 \cdot \left(-\frac{20}{r^2}\right)}{\left(\frac{20}{r}\right)^2} = \frac{40\pi r + 40\pi r}{\frac{400}{r^2}} = \frac{80\pi r}{\frac{400}{r^2}} = \frac{40\pi r^3}{100} = \frac{40\pi r^3}{100}$$

$$\frac{dA}{dr} = 4 + \frac{40r}{(r^2)^2}$$

$$\frac{40r}{(r^2)^2} = -4$$

$$\frac{40}{r^2} = -4$$

$$r^2 = -10$$

$$r = 12.65 \text{ (2dp)} > 0 \quad > 0 = \text{minimum}$$

$$\therefore r = 12.65 \text{ minimum occurs}$$

$$\text{b) i) } \sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$$

hence

$$\sec^2 x = \tan x$$

$$\tan x = \frac{\sin x}{\cos x}$$

and

$$\sec x \tan x = \frac{1}{\cos x}$$

$$\therefore \sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$$

$$\text{ii) } \sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$$

$$\frac{1}{1 - \sin x} = \frac{1 + \sin x}{\cos^2 x}$$

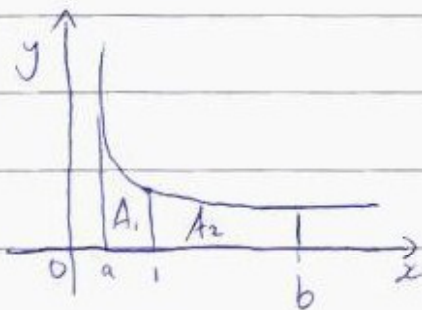
$$\frac{1}{\sin x \cos^2 x} \leftarrow \sin x \cos^2 x = 1 - \sin x$$

$$\text{hence } \therefore \sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$$

$$\begin{aligned}
 \text{iii) } & \int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x} dx \\
 & = \int_0^{\frac{\pi}{4}} -\sin x dx \\
 & = (\cos x) \Big|_0^{\frac{\pi}{4}} \\
 & = \cos \frac{\pi}{4} - \cos 0 \\
 & = \cos \frac{1}{\sqrt{2}} - 1
 \end{aligned}$$

≠

$$c) y = \frac{1}{x} \quad x > 0$$



A_1 and $A_2 = 1 \text{ unit}^2$

$$\int_a^1 \frac{1}{x}$$

$$= (\log_e x) \Big|_a^1$$

$$= \log_e 1 - \log_e a$$

$$= 0 - \log_e a$$

$$a = \log_e$$

$$\int_1^b \frac{1}{x}$$

$$= (\log_e x) \Big|_1^b$$

$$= \log_e b - \log_e 1$$

$$= \log_e b - 0$$

$$b = -\log_e$$

Additional writing space on back page.