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(a)(i) $T_n = a + (n-1)d$

$a = 1000$

$n = 9$

$d = 750$

$T_9 = 1000 + (9-1)750$

$= 7000m$ or $7km$.

(iii) $T_n = 10000$

$a = 1000$

$n = ?$

$d = 750$

$10000 = 1000 + (n-1)750$

$10 = (n-1)750$

$13\frac{1}{3} = 1000(n-1)$

In the 13th week she will run 10km.

(iii)

$1000 + 1750 + 2500 + 3250 + 4000 + 4750 + 5500 + 6250 + 7000 + 7750 + 8500 + 9250 + 10,000 + 10^2$

* In 26 weeks she will run a total distance of 71600m or 71.6km

(b) $\int_0^2 e^{2x} \cdot dx - \int_0^2 e^{-x}\, dx$

$20 \int_0^2 e^{2x} - e^{-x}\, dx$

$= 20 \left[2e^{2x} \right]$

$= 20 \left[\frac{1}{2} e^{2x} - e^{-x} \right]_0^2$

$= 20 \left(\frac{1}{2} e^4 - e^{-2} \right) - \left(\frac{1}{2} - 1 \right)$

$= 553.2247\ 947$

$\doteq 553.22$ units^2.

(c) (i) $P(\text{two mint centres}) = \frac{4}{12} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{36}$.

(ii) $P(\text{same centre}) = \left(\frac{4}{12}\right)^2 \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{108}$.

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(iii) $P(\text{diff centres}) = \frac{1}{3} \times \left(\frac{1}{4}\right)^3 \times \left(\frac{1}{4}\right)^3$

$= \frac{1}{12288}$

(d) $f(x) = 1 + e^x$

$(1 + e^x) \times f(1 + e^x) = (1 + e^x) + (-1 + e^x)$

~~$LHS = -1 + e^{x^2}$~~

~~$RHS = e^{2x}$~~

~~$\therefore f(x) \times f(x) = f(x) + f(x)$~~

$LHS = (1 + e^x) - (1 + e^x)$

$= -1 + e^x - e^x - e^{x^2} \quad = -1 - e^{x^2}$

$RHS = (1 + e^x) * - (1 + e^x)$

$= -1 + e^x - e^x - e^{x^2}$

$= -1 - e^{x^2}$

$\therefore LHS = RHS$

$\therefore f(x) \times f(-x) = f(x) + f(-x)$