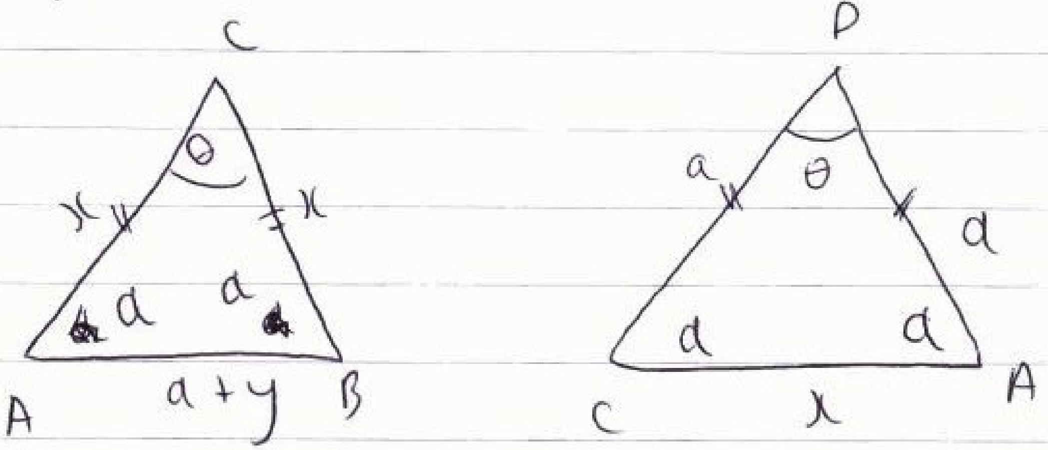
Start here for
Question Number: 10

a) i) ΔABC



$\angle A$ is common

~~$\angle CBA = d$ (base angles isos Δs)~~

$\therefore \angle CBA = d$ (base angles of isosceles triangle are equal)

$\therefore \angle DCA = d$ (base angles of isosceles triangle are equal)

$\therefore \Delta ABC \,|||\, \Delta ACD$ (Equiangular)

ii) $\cancel{\frac{x}{\sin\alpha}} =$ $x^2 = a^2 + ay$

$$\frac{x}{a} = \frac{a+y}{x}$$ (similar triangles ∴ corresponding sides in similar triangles are equal)

$x^2 = a(a+y)$

$x^2 = a^2 + ay$

iii) $y = a(1 - 2\cos\theta)$

$(a+y)^2 = x^2 + x^2 - 2 \times x \times x \times \cos\theta$

$\cancel{a^2 + 2ay + y^2 = 2x^2 - 2x^2\cos\theta}$

$x^2 = a^2 + ay$

$ay = x^2 - a^2$

$y = \frac{x^2}{a} - a$

$\cancel{y^2 = 2x^2 - 2x^2\cos\theta - a^2 - 2ay}$

$\cancel{x^2(2 - 2\cos\theta) + a(a + 2y)}$

$(a+y)^2 = 2x^2 - 2x^2\cos\theta$

$(a+y)^2 = x^2(2 - 2\cos\theta)$

$(a+y)^2 = (a^2 + ay)(2 - 2\cos\theta)$

$(a+y)^2 = 2a^2 - 2a^2\cos\theta + 2ay - 2ay\cos\theta$

$a^2 + 2ay + y^2 = 2a^2 - 2a^2\cos\theta + 2ay - 2ay\cos\theta$

$y^2 = 2a^2 - 2a^2\cos\theta + \cancel{2ay} - 2ay\cos\theta - a^2 \cancel{- 2ay}$

$y^2 = a^2 - 2a^2\cos\theta - 2ay\cos\theta$

$\cancel{y^2 + 2ay\cos\theta = a^2 - 2a^2\cos^2\theta}$

$\cancel{y^2 + y = a^2 - 2a^2\cos^2\theta}$

$\cancel{2a\cos\theta}$

$\cancel{y^2 = a^2 - 2a^2\cos\theta - 2a(}$

$\therefore y^2 = a^2(1 - 2\cos\theta)$

$\therefore y = a(1 - 2\cos\theta)$

Additional writing space on back page.

i) $x^2 = a^2 + ay$ $\qquad y = a(1 - 2\cos\theta)$

$y \leq 3a$ $\qquad a(1 - 2\cos\theta)$

$x^2 = a^2 + ay$ $\qquad -1 \leq a(1 - 2\cos\theta) \leq 1$

$\cancel{ay = x^2 - a^2}$ $\qquad -\frac{1}{a} \leq 1 - 2\cos\theta \leq 1/a$

$\cancel{y = \frac{x^2}{a} - a}$

$\cancel{\frac{x^2}{a} - a = a(1 - 2\cos\theta)}$ $\qquad \therefore y \leq 3a$

iv) ?

iii) $(a + y^2)^2 = x^2 + x^2 - 2x^2\cos\theta$

$(a + y^2) = 2x^2 - 2x^2\cos\theta$

$(a + y)^2 = 2(a^2 + ay) - 2(a^2 + ay)\cos\theta$

$(a + y)^2 = 2a^2 + 2ay - (2a^2 + 2ay)\cos\theta$

~~a+y~~

$a^2 + 2ay + y^2 = 2a^2 + \cancel{2ay} - (2a^2 + 2y)\cos\theta - a^2 - \cancel{2ay}$

$y^2 = a^2 - (2a^2 + 2ay)\cos\theta$

Start here for Question Number: 10

b) $x^2+y^2=r^2$ $B=(r,0)$

i) $V=\pi\int y^2$ $y^2=r^2-x^2$

$\pi\int^r r^2-x^2$ $A=$ P

$\pi\int^r$

~~Let OB = r~~

$OB=r$

$OA=$

= Volume of sector ~~OB~~ integral OB − ~~secto~~ volume of sector OPX − Volume Δ OPA

$\pi\int_0^r r^2-x^2 \quad -\pi\int_0^{A} \text{sector OPX} \quad -\pi\int_0^{PA} \text{triangle OAP}$??

Hemisphere $=\frac{4}{3}\pi r^3$

$\sin\theta=\frac{OA}{r}$

$\boxed{\frac{4}{3}\pi r^3}$ −

$OA=r\sin\theta$

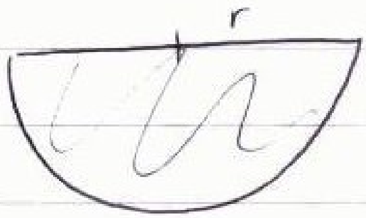
$\therefore AP=r^2-(r\sin\theta)^2$

$AP^2=r^2-r^2\sin^2\theta$

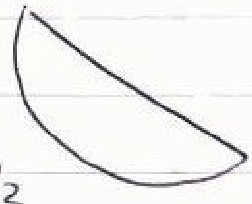
~~$AP=r-r\sin\theta$~~

$AP=\sqrt{r^2-r^2\sin^2\theta}$??

iii) radius = r



V of sphere = $\frac{4}{3}\pi r^3$



V of hemisphere = $\frac{4}{3}\pi r^3 \times \frac{1}{2}$

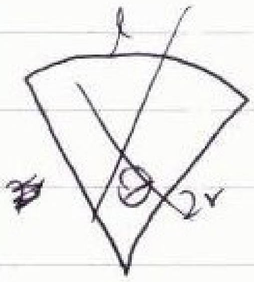
$= \frac{2}{3}\pi r^3 = \cancel{F}\ 1$

$2\pi r^3 = 3$

$\pi r^3 = 3/2$

$r^3 = \frac{3}{2\pi}$

$r = \sqrt[3]{\frac{3}{2\pi}}$



$l = r\theta$

$\therefore h =$

$\frac{2}{3}\pi r^3 = 1/2$

$2\pi r^3 = 1\frac{1}{2}$

$\pi r^3 = \frac{3}{4}$

$r^3 = \frac{3}{4\pi}$

$r = \sqrt[3]{\frac{3}{4\pi}}$

??

Additional writing space on back page.