Question 6

a) $y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

x-int $y = 0$

$y = \sqrt{4 - 0^2}$

$= \pm 2$

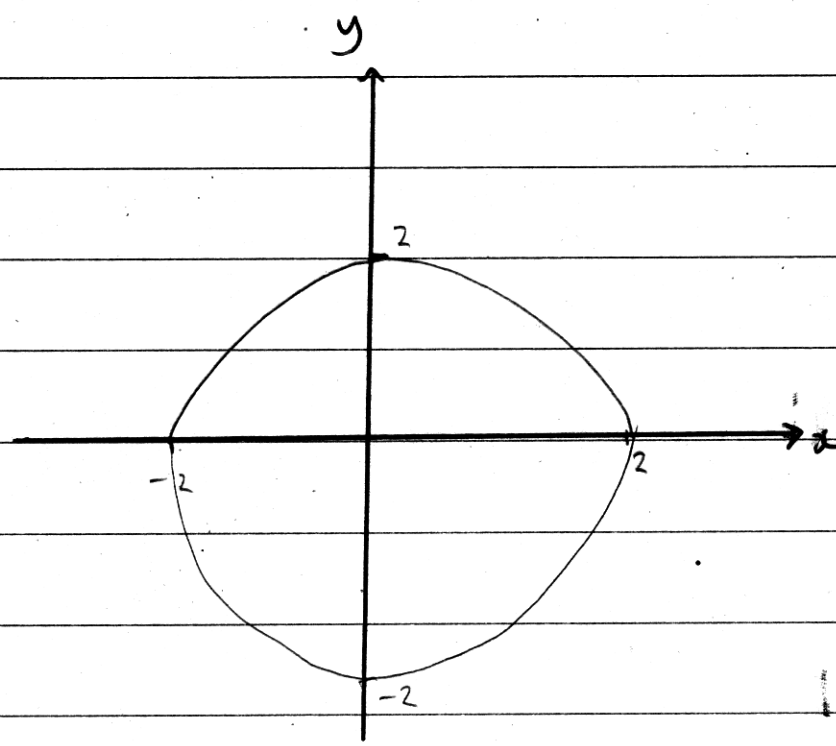
y-int $x = 0$

$0 = \sqrt{4 - x^2}$

$0 = 4 - x^2$

$x^2 = 4$

$x = \pm 2$



$\therefore$ range $-2 \leq y \leq 2$

b i) $f'(x) = 3(x+1)(x-3)$

$f(x) = \int 3(x^2 - 3x + x - 3)\, dx$

$= \int 3x^2 - 6x - 9 \, dx$

$= \frac{3x^3}{3} - \frac{6x^2}{2} - 9x + C$

$= x^3 - 3x^2 - 9x + C$

$(0, 12)$

$12 = 0^3 - 3(0)^2 - 9(0) + C$

$\therefore C = 12$

$\therefore y = x^3 - 3x^2 - 9x + 12$

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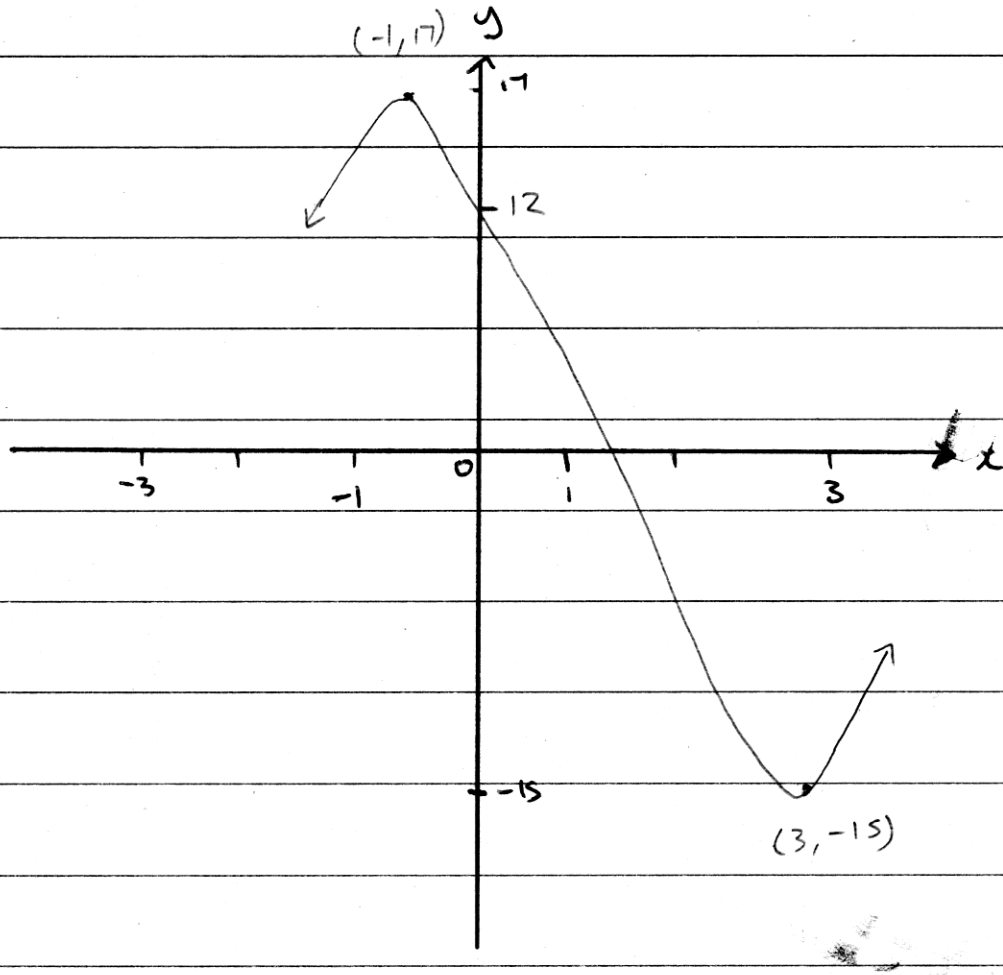
ii)



iii) $f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$6x - 6 = 0$

$6x = 6$

$x = 1$

$\therefore$ concave up for all $x > 1$

c) $V = \pi \int_a^b x^2 \, dy$

$y = \frac{x^4}{4}$

$4y = x^4$

$\sqrt{4y} = x^2$

$= \pi \int_0^4 (4y)^{\frac{1}{2}} \, dx$

$= \pi \left[\frac{4y^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$

$= \pi \left[\frac{4(4)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4(0)^{\frac{3}{2}}}{\frac{3}{2}} \right]$

$= \pi \left[\frac{64}{3} \right]$

$\therefore V = \frac{64\pi}{3}$ units^3