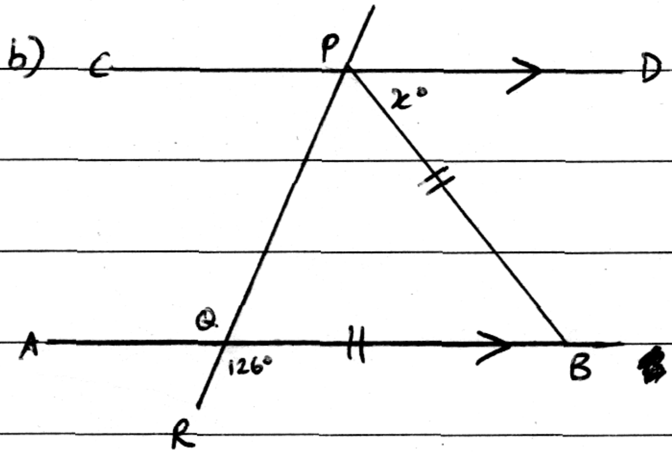
Question 3

a) Value $= 1000\left(1+\frac{3.5}{100}\right)^{20}$

$= 1000(1.035)^{20}$

$= \$1989.79$ (to the nearest cent)

b)



$\angle BQR + \angle BQP = 180°$

(adjacent angles on a straight line are supplementary)

$126° + \angle BQP = 180°$

$\therefore \angle BQP = 54°$

$\angle BQP = \angle BPQ$ (~~sides opp~~

(angles opposite equal sides of a triangle)

$\therefore \angle BPQ = 54°$

$\angle BQR = \angle DPQ$ (corresponding angles on parallel lines $CD \parallel AB$)

$\therefore \angle DPQ = 126°$

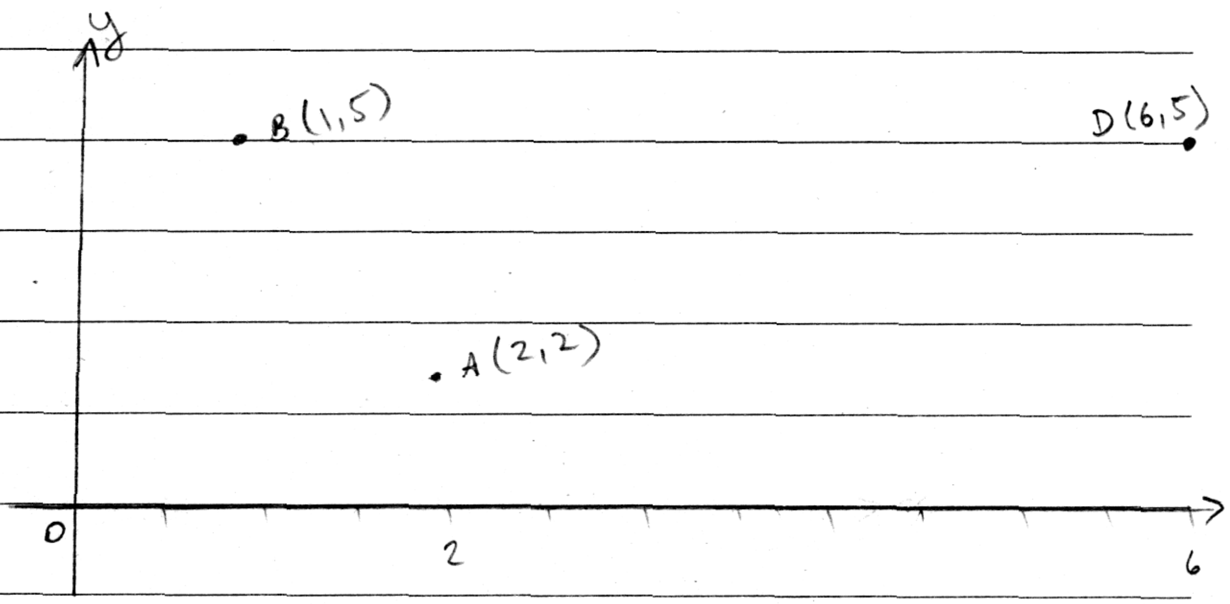
$\angle DPQ = x^\circ + \angle BPQ$

$126^\circ = x^\circ + 54^\circ$

$\therefore x^\circ = 126^\circ - 54^\circ$

$= 72^\circ$

c)



i) $M_{AB} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{2+1}{2}, \dfrac{2+5}{2}\right)$

$= \left(\dfrac{3}{2}, \dfrac{7}{2}\right)$

ii) $m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-2}{1-2}$

$= \frac{3}{-1}$

$m_{AB} = -3$

$m_1 \times m_2 = -1$ for perpendicular lines

$-3 \times m_2 = -1$

$m_2 = \frac{-1}{-3}$

$= \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - \frac{7}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$

$y - \frac{7}{2} = \frac{x}{3} - \frac{3}{6}$

$= \frac{x}{3} - \frac{1}{2}$

$2y - 7 = \frac{2x}{3} - 1$

$6y - 21 = 2x - 3$

$0 = 2x - 6y + 18$

$0 = x - 3y + 9$

eqn of perpendicular bisector as required.

~~iii) DAC = DBC~~

~~$(DAC)^2 = (DBC)^2$~~

~~∴~~

iii) DAC = DBC   since C lies on the y axis, x co-ordinate is 0.

$(DAC)^2 = (DBC)^2$

$\therefore (0-2)^2 + (y-2)^2 = (1-0)^2 + (5-y)^2$

$4 + (y-2)^2 = 1 + (5-y)^2$

$4 + (y^2 - 2y + 4) = 1 + (25 - 10y + y^2)$

$4 + \cancel{y^2} - 2y + 4 = 1 + 25 - 10y + \cancel{y^2}$

$8y = 18$

$y = \frac{18}{8}$

$= \frac{9}{4}$

∴ co-ordinates of C are $(0, \frac{9}{4})$

iv) $y = 5$ —— (1)

$x - 3y + 9 = 0$ —— (2)

sub (1) into (2)

$x - 3(5) + 9 = 0$

$x - 15 + 9 = 0$ ∴ co-ordinates of D are $(6, 5)$

$x = 6$

v) Since D lies on the perpendicular bisector we are able to find the perpendicular distance between AB and D

$$\text{Perpendicular Distance} = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$D(6, 5)$$

equation of line AB $\Rightarrow y - y_1 = m(x - x_1)$

$$y - 5 = -3(x - 1)$$

$$y - 5 = -3x + 3$$

$$3x + y - 8 = 0$$

equation of AB

$$\therefore \text{Perpendicular distance} = \left| \frac{3 \times 6 + 1 \times 5 - 8}{\sqrt{3^2 + 1^2}} \right|$$

$$= \left| \frac{18 + 5 - 8}{\sqrt{10}} \right|$$

$$= \frac{15}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{15\sqrt{10}}{10} = \frac{3\sqrt{10}}{2}$$

$$D_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-2)^2 + (5-2)^2}$$
$$= \sqrt{(-1)^2 + (3)^2}$$
$$= \sqrt{1+9}$$
$$= \sqrt{10}$$

Area of $\triangle ABD = \frac{1}{2}bh$

$$= \frac{1}{2} \times \sqrt{10} \times \frac{3\sqrt{10}}{2}$$

$$= \frac{30}{4}$$

$$= \frac{15}{2} \text{ units}^2.$$