BOARD OF STUDIES
NEW SOUTH WALES

Q 2

(a) tangent to $y = e^{2x}$ at $(0, 1)$

$y = e^{2x}$

$y' = 2e^{2x}$

when $x = 0$

$y' = 2e^{0}$

$= 2$

$\therefore m = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 0)$

$y - 1 = 2x$

$\therefore 2x - y + 1 = 0$ $\quad [\, y = 2x + 1 \,]$

(b) (i) $y = x \sin x$

$y' = vu' + uv'$

$= \sin x + x \cos x$

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(b) (ii) $y = \frac{\ln x}{x^2}$

$y' = \frac{vu' - uv'}{v^2}$

$\cancel{\left(= x^2 \cdot \frac{1}{x} - 2\right)}$

$= \frac{x^2 \cdot \frac{1}{x} - \ln x \cdot 2x}{(x^2)^2}$

$= \frac{\frac{x^2}{x} - 2x \ln x}{x^4}$

$= \frac{x - 2x \ln x}{x^4}$

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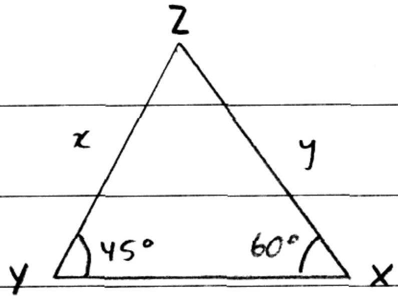
(c)

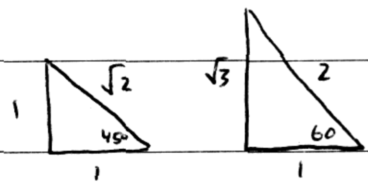


$\left(\cancel{\dfrac{x}{y} =}\right)$

Using the sine rule;

$$\frac{x}{\sin X} = \frac{y}{\sin Y}$$

$$\frac{x}{\sin 60^\circ} = \frac{y}{\sin 45^\circ}$$



$$\frac{x}{\frac{\sqrt{3}}{2}} = \frac{y}{\frac{1}{\sqrt{2}}}$$

$$\cancel{\frac{2x}{\sqrt{3}} = \frac{\sqrt{2}\,y}{1}}$$

$$\cancel{2x = \sqrt{6}\,y}$$

$$\cancel{x = \frac{\sqrt{6}\,y}{2}}$$

$$\cancel{\therefore \frac{x}{y} = \frac{\sqrt{6}}{2}}$$

$$\frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$$

$$\frac{x}{y} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{1}$$

$$\therefore \frac{x}{y} = \frac{\sqrt{6}}{2}$$

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(d) (i) $\int \cos 3x \, dx = \frac{1}{3} \sin 3x + c$

(ii) $\int_0^1 (e^{5x} - 1) \, dx = \left[\frac{e^{6x}}{6} - x \right]_0^1$

$= \left[\left(\frac{e^6}{6} - 1 \right) - \left(\frac{e^0}{6} - 0 \right) \right]$

$= \left[\left(\frac{e^6}{6} - 1 \right) - \frac{1}{6} \right]$

$= \left[\frac{e^6}{6} - \frac{5}{6} \right]$

$= \frac{e^6 - 5}{6}$

$\doteqdot 66 \cdot 4$ (1 d.p.)