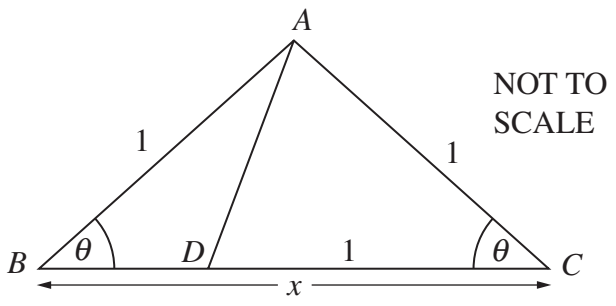
Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, ABC is an isosceles triangle where $\angle BAC = \frac{3\pi}{5}$ and $AB = AC = 1$. The point D is chosen on BC such that $CD = 1$.

Let $BC = x$, and let $\angle ABC = \theta$, and note that $\theta = \frac{\pi}{5}$.

(i) Show that $\angle ADC = 2\theta$ and hence show that triangles DBA and ABC are similar. **3**

(ii) From part (i) deduce that $x^2 - x - 1 = 0$. **1**

(iii) By using the cosine rule, deduce that **2**

$$\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}.$$

(b) When a valve is released, a chemical flows into a large tank that is initially empty. The volume, V litres, of chemical in the tank increases at the rate

$$\frac{dV}{dt} = 2e^{t} + 2e^{-t}$$

where t is measured in hours from the time the valve is released.

(i) At what rate does the chemical initially enter the tank? **1**

(ii) Use integration to find an expression for V in terms of t. **2**

(iii) Show that $2e^{2t} - 3e^{t} - 2 = 0$ when $V = 3$. **1**

(iv) Find t, to the nearest minute, when $V = 3$. **2**